

International Conference

PARALLEL COMPUTER ALGEBRA '2010

Tambov, Russia
June 29 – July 3, 2010

Tambov State University named after G.R. Derzhavin

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Topics:

Parallel algorithms for computer algebra

Software techniques for parallel computer algebra systems

Applications of parallel computer algebra in all fields

Use of computer algebra to design parallel algorithms or software in other areas

The International Conference

PARALLEL COMPUTER ALGEBRA

June 29 - July 3, 2010, Tambov, Russia

Tambov State University named after G.R. Derzhavin

June 29 Tuesday

- 13:00 **Bus from Hotel**
13:30 **Dinner**
14:00 **Tambov bus Excursion**
18:00 **Supper**

June 30 Wednesday

- 9:00–9:45 **Registration** (TSU, Komsomolskaya sq. 5, bus N18 from train station)
9:45–10:00 **Opening the conference**
10:00–10:45 **Anatoly Panyukov, Vasily Gorbik.** Exact and Guaranteed Accuracy Solutions of Linear Programming Problems by Distributed Computer Systems with MPI
10:45–11:30 **Gennadi Malaschonok.** Fast matrix decomposition in parallel computer algebra
11:30–12:00 **Coffee break**
12:00–12:30 **Oksana Pereslavl'tseva.** Parallel algorithms for computing the characteristic polynomials based on the method of homomorphic images
12:30–13:00 **Alexey Pozdnykin.** Out-of-core parallel polynomial arithmetic
13:00–15:00 **Dinner**
15:00–15:45 **Philippe Aubry and Annick Valibouze.** Parallel computation of Lagrange resolvents by multi-resolvents
15:45–16:30 **Dmitry Pavlov.** Parallelized computation of Extended Universal Groebner Basis
16:30–17:00 **Coffee break**
17:00–17:30 **Oleg Yaremko, Natalia Yaremko.** Parallel computing for the Fourier transform with discontinuous coefficients
17:30–18:00 **Irina Shlykova, Arcady Ponosov, Yuriy Nepomnyashchikh, Andrei Shindiapin.** Stability analysis around singular stationary points and computer algebra systems
18:30 **Supper**

July 1 Thursday

- 10:00–10:45 **Alexey Lapaev.** DFT for polynomials in parallel algorithms
- 10:45–11:30 **Mikhail Cherepniov.** Some estimations of performance of parallel algorithms for solving large linear systems over $\text{GF}(2)$
- 11:30–12:00 **Coffee-break**
- 12:00–12:30 **Vasily Astakhov.** Estimates of the running time and memory requirements of the new algorithm of solving large sparse linear systems over field with two elements
- 12:30–13:00 **Natasha Malaschonok.** A parallel algorithm for symbolic solving partial differential equations
- 13:00–15:00 **Dinner**
- 15:00– 15:30 **Andrey Betin.** The parallel algorithm for calculation of an adjoint matrix and experiments
- 15:30–16:00 **Maxim Starov.** One approach for computation of Groebner Basis
- 16:00–16:30 **Coffee-break**
- 16:30–17:00 **Vasyl Tereshchenko.** Parallel computing in problems of geometric modeling
- 17:00–17:30 **Stephen Watt.** An Analytic Model for Colluding Processes
- 18:00 **Freandship supper**

July 2 Friday

- 9:00 **Excursion to Rakhmaninov Museum in Ivanovka**

July 3 Saturday

- 10:00–10:45 **Vladimir Kornyak.** Constructive Study of Finite Quantum Models
- 10:45–11:30 **Yuri Blinkov.** On parallel computation of the bifurcation points for the logistic map
- 11:30–12:00 **Coffee-break**
- 12:00–12:30 **Yakov Romm, Garik Dzhanunts.** Difference-polynomial solutions of Cauchy problem for the ordinary differential equations using the parallel recovery coefficients of the polynomial from its roots
- 12:30–13:00 **Michail Maidakov.** Parallel computing of discrete Sibsons interpolation on GPU
- 13:00–15:00 **Dinner**
- 15:00–15:15 **Shamil Otsokov.** A method of error-free computation in residue number system
- 15:45–16:00 **Gennadi Malaschonok.** Parallel computer algebra system ParCA. Main characteristics
- 16:00–16:30 **Coffee-break**
- 16:30–17:00 **Andrey Betin.** Parallel computer algebra system ParCA. Demonstration
- 17:00–17:30 **Poster session.** Selection of the best student work
- 18:00 **Supper**

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Estimates of the running time and memory requirements of the new algorithm of solving large sparse linear systems over the field with two elements

Vasily Astakhov

Moscow State University, Moscow, Russia

e-mail: astvvas89@mail.ru

A new algorithm of solving large sparse linear systems over field with two elements is considered in this work. Algorithm was proposed by M.A. Cherepniov. Algorithm uses the construction of matrix Pade approximations over finite fields. It is supposed that elements of approximation polynomials are independent and are identically distributed. Method for finding distributions of coranks of random symmetric, antisymmetric and common matrices is constructed. Lower and upper bounds for number of previous approximations sufficient to construct the next one are obtained. The logarithmic dependence for sufficient number of keeping approximations on every step for successful completion of algorithm with probability of 0.99 is found. Using the computer program exact values of estimates of running time and memory requirements are found, results are given in this work.

Parallel computation of Lagrange resolvents by multi-resolvents

Philippe Aubry, Annick Valibouze

Université Pierre et Marie Curie, Paris, France

e-mail: philippe.aubry@upmc.fr, annick.valibouze@upmc.fr

The goal of this paper is the parallel computation of Lagrange resolvents of a univariate polynomial. The computation of Lagrange resolvents of a univariate polynomial has significance for Galois theory. Since Lagrange's algorithms, many other algorithms for computing some particular resolvents, called absolute, were developed from the fundamental theorem of symmetric functions. The algebraic algorithms for non absolute resolvents are few and recent because they use galoisian ideals that were introduced recently. However these algorithms become time and space consuming when the degree of the polynomial increases. This motivates their parallelisation. In 2004 N. Rennert presented a multimodular method for computing absolute resolvents of polynomials with integer coefficients. We show that the same techniques can be extended to any resolvent. This method is

naturally parallelisable. Moreover, we give a decomposition formula of resolvents which makes possible another level of parallelisation. This leads to an algorithm with a doubly parallel character.

The parallel algorithm for calculation of an adjoint matrix and experiments¹

Andrey Betin

Tambov State University, Tambov, Russia

e-mail: `Andrey_Betin@mail.ru`

The adjoint matrix is the transposed matrix of algebraic additions. If the matrix determinant is invertible, that the inverse matrix can be calculated as the adjoint matrix, with division by the determinant.

Following decomposition on a multiplier of a inverse matrix is known. if $\mathfrak{A} = \begin{pmatrix} A & C \\ B & D \end{pmatrix}$ – invertible matrix and A invertible block, then inverse matrix \mathfrak{A}^{-1} has the following decomposition:

$$\begin{bmatrix} (I & -A^{-1}C) \\ 0 & I \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & (D - BA^{-1}C)^{-1} \end{bmatrix} \begin{bmatrix} I & 0 \\ -B & I \end{bmatrix} \begin{bmatrix} A^{-1} & 0 \\ 0 & I \end{bmatrix}.$$

Application of determinant identities allows to calculate the adjoint matrix by means of similar decomposition of the adjoint matrix (Malaschonok G.I. Matrix method of calculation in the commutative ring. The monography. Tambov: Publishing house TSU of G.R.Derzhavin, 2002. 214 with. 78-82.). On the basis of this algorithm parallel algorithm is written. Experiments with parallel algorithm on a supercomputer MVS100k of Joint Supercomputer Center of the RAS were made. Supercomputer consist of 1184 calculation modules, each module consist of two IntelXeon 3Ghz four core processors and 8Gb of internal memory.

In experiments are used matrices an order 512×512 , 1024×1024 , 2048×2048 and 4096×4096 .

For an estimation of efficiency of parallel algorithms we will enter the following concept. Let t_k is the computation time of the algorithm for the cluster with k processors. At transition from the cluster with n processors to the cluster with k processors, $k > n$, the efficiency is equal 100%, When $t_n/t_k = k/n$. The efficiency is equal to zero, when $t_k = t_n$. To define an

¹Supported by the Sci. Program Devel. Sci. Potent. High. School, RNP 2.1.1.1853.

efficiency of computations at other values t_n/t_k we will define the efficiency as time function.

$$\alpha_{n,k} = \frac{t_n/t_k - 1}{k/n - 1} \cdot 100\%.$$

Efficiency of calculations appears for matrices which sizes more than 512×512 and with growth of the sizes of matrices parallel calculations are put more more effectively, but with problem growth there is a problem of shortage of operative memory. For the decision of the given problem it is possible to store matrices not in operative memory, but in the form of files on a hard disk, reading out only the necessary matrix blocks. The size of the maximum block gets out taking into account accessible operative memory of knot. Experiments with parallel multiplication have been made Such matrices which have shown that with growth of number of processors efficiency of calculations is from 30 % to 100 % that speaks about efficiency parallelization.

On parallel computation of the bifurcation points for the logistic map¹

Yuri A. Blinkov

*Department of Mathematics and Mechanics,
Saratov State University,
410012 Saratov, Russia*

e-mail: BlinkovUA@info.sgu.ru

The logistic map is a discrete-time analogue of the continuous logistic Verhulst's equation and has the form of a one-parameter nonlinear recurrence relation

$$x_{n+1} = \mu x_n(1 - x_n), \quad \mu > 0. \quad (1)$$

Detecting the bifurcation values of parameter μ is a hard computational problem [2]. It is described by the following system of polynomial equations

$$\begin{cases} x_2 = \mu x_1(1 - x_1), \\ x_3 = \mu x_2(1 - x_2), \\ \dots \\ x_n = \mu x_{n-1}(1 - x_{n-1}), \\ x_1 = \mu x_n(1 - x_n), \\ \mu^n \prod_{k=1}^n (1 - 2x_k) = 1. \end{cases} \quad (2)$$

¹Work partially supported by grant 0-01-00200 from the Russian Foundation for Basic Research and by grant 3810.2010.2 from the Ministry of Education and Science of the Russian Federation.

From the system (2) one can compute a polynomial in μ by doing elimination of the variables $\{x_1, \dots, x_n\}$. This can be achieved by computing a Gröbner basis for a degree compatible term order and then by constructing a univariate polynomial in μ that belongs to the ideal generated by the system (2). The last construction is done by the linear algebra methods applied to the ideal as a vector space generated by the power products in $\{x_1, \dots, x_n, \mu\}$. In doing so, computation of roots of the univariate polynomial obtained is a much more simple problem to detect the values of μ directly from the multivariate system (2).

We discuss the algorithmic aspects of this approach the specialized computer algebra system *GINV* [1] to compute the bifurcation polynomial in μ for $n = 9$. Among the real roots of this polynomial we found the bifurcation point $\mu = 3.687196\dots$

References

- [1] Yu. A. Blinkov and V. P. Gerdt. Specialized computer algebra system *GINV*. *Programming and Computer Software*, 34(2):112–123, 2008. <http://invo.jinr.ru/ginv/index.html>
- [2] I. S. Kotsireas and K. Karamanos. Exact computation of the bifurcation point b_4 of the logistic map and the bailey-broadhurst conjectures. *Journal Bifurcation and Chaos*, 14:2417–2423, 2004.

Some estimations of performance of parallel algorithms for solving large systems over $\text{GF}(2)$

Mikhail Cherepnev

Moscow State University, Moscow, Russia

e-mail: Cherepniiov@gmail.com

This topic explains how to estimate the running time and RAM volume required by programs of Wiedemann-Coppersmith algorithm, Montgomery’s algorithm, some modifications of them and new algorithm when uploading multiple compute nodes and some other details of these algorithms.

Constructive Study of Finite Quantum Models

Vladimir V. Kornyak

*Laboratory of Information Technologies Joint Institute for Nuclear
Research, 141980 Dubna, Russia*
e-mail: kornyak@jinr.ru

Evolution of any quantum system is described by unitary operators forming an unitary group. We define a *finite quantum model* as a quantum system which evolution operators form a finite subgroup of general unitary group. General properties of unitary representations of finite groups allow to embed *any quantum system* with a finite group of evolution operators into a larger *classical system* with simple permutational dynamics that can be efficiently simulated by a computer. Possible applications of finite quantum models include, in particular: (a) study of different models in *quantum computing*; (b) various models in *particle physics* with finite symmetry groups, like, e.g., non-Abelian flavor symmetries of quarks and leptons.

DFT for polynomials in parallel algorithms¹

Aleksey Lapaev

Tambov State University, Tambov, Russia
e-mail: alapaev@gmail.com

We investigate sequential and parallel algorithms for polynomial arithmetic based on discrete Fourier transform (DFT). Algorithms for polynomial multiplication are discussed. Sequential algorithms for polynomial matrix are proposed. Each algorithm based on DFT has been compared with similar algorithm based on Chinese remainder theorem. In the last part of work parallel algorithms for calculation DFT and multivariable polynomials multiplication are considered. Theoretical expressions of complexity are presented for each algorithm. Results of experiments on MVS cluster are presented for parallel algorithms.

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Parallel computing of discrete Sibsons interpolation on GPU

Michail Maidakov

Belgorod State University, Belgorod, Russia

e-mail: m-email@inbox.com

This paper presents a method for parallel computation of discrete Sibson interpolation without the prior calculation of Voronoi diagram. The comparison of the computation time of the proposed method using parallel processing on the GPU and the serial on the CPU. Acceleration of the method is achieved by using simple data structures to store intermediate information and parallel computing on the GPU.

Fast matrix decomposition in parallel computer algebra¹

Gennadi Malaschonok

Tambov State University, Tambov, Russia

e-mail: malaschonok@ya.ru

The new algorithms for finding matrix decomposition and matrix inversion in arbitrary fields are described. For the commutative domains the algorithm for finding adjoint matrices is proposed. These algorithms have the same complexity as matrix multiplication and do not require pivoting. For singular matrices they allow to obtain a nonsingular block of the biggest size. The proposed algorithms are pivot-free, and do not change the matrix block structure. They are suitable for parallel hardware implementation.

A parallel algorithm for symbolic solving partial differential equations²

Natalia Malashonok

Tambov State University, Tambov, Russia

e-mail: namalaschonok@gmail.com

A parallel algorithm for symbolic solving partial differential equations by means of Laplace–Carson transform is produced. The problem is reduced to solving linear algebraic systems with polynomial coefficients, for

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which efficient parallel algorithms exist. It permits to construct a fast parallel algorithm for systems of partial differential equations. An algorithm includes a procedure to obtain compatibility conditions for initial data.

A method of error-free computation in residue number system

Shamil Otsokov

Moscow Power Energetic University, Moscow, Russia

e-mail: `shamil24@mail.ru`

In article the method exact computation in residue number system is offered. It allows to expand essentially a range of representation of binary fractional numbers in comparison with existing method. New presentation format of binary fractions in a modular number system is offered.

Exact and Guaranteed Accuracy Solutions of Linear Programming Problems by Distributed Computer Systems with MPI

Anatoly Panyukov, Vasily Gorbik

South Ural State University, Chelyabinsk, Russia

e-mail: `a_panyukov@mail.ru`, `vgorbik@gmail.com`

Techniques of obtaining both exact and guaranteed accuracy solutions of linear programming problems and methods of increasing accuracy of computations by distributed computer systems with MPI are subjects of this paper. To obtain the solutions the rational and arbitrary precision floating point interval arithmetic libraries are applied. Methods of adaptation of the used data types to MPI are presented. Results of computational experiments based on introduced parallel versions of algorithms for solving systems of linear equations and linear programming problems demonstrate effectiveness of their application.

Parallelized computation of Extended Universal Gröbner Basis¹

Dmitry Pavlov

Saint-Petersburg State Polytechnical University, Saint-Petersburg, Russia
e-mail: `dmitry.pavlov@gmail.com`

The article presents an algorithm to calculate Extended Universal Gröbner Basis (EUGB), working on wide range of polynomial ideals. The EUGB(\mathfrak{A}) of a polynomial ideal \mathfrak{A} is defined as a finite set of polynomials $\{f_i\}$ whose Young diagrams $Y(f_i)$ meet the following condition: $\dim(\mathcal{L}(Y(f_i)) \cap \mathfrak{A}) = 1$ (where \mathcal{L} denotes the span of a set of polynomials in the quotient algebra of the ideal). It is known that the EUGB contains the Universal Gröbner Basis. The algorithm is based on geometry of Young diagrams in \mathbb{Z}_+^d , and finds the polynomials of EUGB mostly independently, which makes it able to run in parallel. An outline of the parallel version of the algorithm is given.

Parallel algorithms for computing the characteristic polynomials based on the method of homomorphic images²

Oxana Pereslavl'tseva

Tambov State University, Tambov, Russia
e-mail: `Pereclavtseva@rambler.ru`

There are produced parallel algorithms for computing the characteristic polynomials for integer and polynomial dense matrices. The algorithms are based on the method of homomorphic images in the ring of integers and in the ring of polynomials. We have obtained an upper bound for numerical coefficients of a characteristic polynomial. There are stated and discussed results of experiments with parallel algorithms for computing the characteristic polynomials of integer and polynomials matrices. The experiments with parallel algorithm are conducted on cluster MVS100k of Joint Super-Computer Center RAS.

¹I wish to thank Nickolay Vasiliev for encouraging and supervising my work on this topic.

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Out-of-core parallel polynomial arithmetic¹

Alexey Pozdnykin

Tambov State University, Tambov, Russia

e-mail: `pozdalex@mail.ru`

This paper presents the description of structure of polynomials on the external data carrier. The algorithms for addition and parallel multiplication of polynomials are scrutinized. The results of experiments conducted with parallel multiplication of polynomials on cluster are given.

Difference-polynomial solutions of Cauchy problem for the ordinary differential equations using the parallel recovery coefficients of the polynomial from its roots²

Ya. E. Romm, G.A. Dzhannunts

Taganrog State Pedagogical Institute, 347936 Initsiativnaya street, 48,

Taganrog, Russia

e-mail: `romm@list.ru`, `janunts@inbox.ru`

Piecewise polynomial approximation of the Cauchy's problem solution for the ordinary differential equations (DE) is based on Newton's interpolation polynomial. The polynomial coefficients are reconstructed from its roots, using the parallel algorithm. At the current subinterval on difference approximations the right part DE is interpolated, coefficients of the interpolation polynomial are calculated, the solution is approaching with its primitive. The process is repeated iteratively until the approximation error of the right side is minimized. Computer realization has highly accurate approximation of the solution at a low time complexity due to the relatively large size of the difference step.

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Stability analysis around singular stationary points and computer algebra systems

Andrei Shindiapin, Yuriy Nepomnyashchikh
Eduardo Mondlane University, Mozambique
e-mail: shindiapin@yahoo.com, yuvn2@yandex.ru

Irina Shlykova, Arcady Ponosov
Norwegian University of Life Sciences, Norway
e-mail: irinsh@umb.no, arkadi@umb.no

A CAS-based method to study asymptotic properties of solutions to systems of differential equations with distributed time-delays and Boolean-type nonlinearities (step functions) is offered. Such systems arise in many applications, but this paper deals with specific examples of such systems coming from genetic regulatory networks. A challenge is to analyze stable stationary points which belong to the discontinuity set of the system (thresholds). In this work we do the stability analysis around such points in the presence of delays. The basic technical tool consists in replacing step functions by the so-called "logoid functions" and investigating the smooth systems thus obtained.

One approach for computation of Gröbner Basis¹

Maxim Starov
Tambov State University, Tambov, Russia
e-mail: m.starov@mail.ru

One of approaches to parallel calculation of Gröbner's bases is considered. Parallel algorithms of calculation of Gröbner's bases and a step kind of matrices are resulted. Results of experiments are discussed. On cluster with the distributed memory containing of some hundreds of kernels results show good scalability.

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Parallel computing in problems of geometric modeling

Vasyl Tereshchenko

Kyiv, Ukraine

e-mail: `v_ter@ukr.net`

In this paper, we present new approach in geometric modeling and visualization - the common algorithmic space for multitask geometric modeling of complex phenomena and processes. Basis of the approach is a generalized efficient parallel and recursive algorithm, which uses the common algorithmic tools: data structures and a set of procedures. We illustrate the proposed approach on an example of modeling of thermophysical processes in welding.

An Analytic Model for Colluding Processes

Stephen M. Watt

University of Western Ontario, Canada

e-mail: `smwatt@gmail.com`

We develop a quantitative framework in order to understand how OR parallelism can be used to reduce execution times. In this model, tasks may either succeed or fail and any one success completes the problem. We follow Hoare and call these tasks “colluding”. We model the situation where tasks’ execution times are not known in advance, but instead have some probability distribution of execution times. We show how expected serial and parallel execution times can be computed, and demonstrate how parallel execution can give a lower expected execution time than any serial order, even on a single processor. This model can be applied in domains, such as computer algebra, which use algorithms whose execution times cannot be readily predicted by examining the inputs.

Parallel computing for the Fourier transform with discontinuous coefficients

Yaremko O.E., Yaremko N.N.

PSPU, Penza, Russia

e-mail: yaremki@yandex.ru, m.yaremko@gmail.com

The structure of a non-stationary temperature field of semi-infinite composite bar has the form:

$$u_i(t, x) = \frac{1}{2\pi} \int_0^\infty \frac{\exp(-\lambda^2 t)}{\Omega(t)} v_i(t, x) F(\lambda) d\lambda, \quad (1)$$

$$t > 0, \quad l_{i-1} < x < l_j,$$

$$F(\lambda) = \sum_{j=1}^n F_j(\lambda), \quad F_j(\lambda) = \int_{l_{j-1}}^{l_j} v_j(\xi, \lambda) f_j(\lambda) d\lambda,$$

where $f_j(\xi)$ -initial distribution of temperature in j-a layer, $u_i(t, x)$ - temperature distribution in i-a layer at the moment t, $v_i(\lambda, x)$ – a component i of eigenfunction of Fourier operator, $\Omega(\lambda)$ – spectral density.

First step.

The processor P_j calculates the j component of the eigenfunction $v_j(\lambda, x)$ and the spectral function $F_j(\lambda)$.

Second step.

Processors P_1, \dots, P_n transmit data to the processor P_i . Processor P_i calculates the values of temperature in the i-layer by formula (1).